PROBABILITY PARADOXES

Flipping Coins
Imagine you flip a fair, two-sided coin 10 times. Note that there are $2^{10} = 1024$ possible outcomes.

a) Give an example of one possible outcome: 

b) Now, flip an actual coin, and write down your results:

What is the probability of getting at least one head?

What is the probability of getting exactly four heads?

Two Child Problem
Couple A has two children. If you know that one of the kids is a girl, what is the probability that the other child is also a girl?

Couple B also has two children. If you know that the oldest child is a boy, what is the probability that both children are boys? How is this different from the previous problem?

Bob’s Bus Rides
Every night, Bob either visits his mother (downtown) or his girlfriend (uptown). In order to be completely fair, he goes to the bus stop at a random time each night and takes whichever bus arrives first. Each bus arrives every 60 minutes with perfect regularity. Why does Bob only visit his mother twice a month?
To Play or Not to Play

A casino wants to offer a new game in which a fair coin is tossed at each stage. The pot begins at $1 and is doubled every time a head appears. The first time a tail appears, the game ends and the player wins whatever is in the pot. Thus the player wins $1 if a tail appears on the first toss, $2 if a head appears on the first toss and a tail on the second, $4 if a head appears on the first two tosses and a tail on the third, and so on. What would you be willing to pay the casino for entering the game?

The Prisoners’ Paradox

Captain Hook imprisons Wendy, John and Michael. Tonight, he will choose one of them randomly to be executed, and the other two will be released. Wendy appeals to a guard to tell her the name of one brother to be released. Her argument is that no harm will be done because she already knows that at least one of her brothers will be released, so her chances of being the one executed (\(\frac{1}{3}\)) will not change. The guard refuses, saying that given the new information, there will be only 2 candidates, so her chances will increase to \(\frac{1}{2}\). Who is right?

Challenge Problems

1. If you toss a fair coin 10 times, what is the probability that no four heads appear consecutively?

2. Suppose Couple C has a 25% chance of having a girl and a 75% chance of having a boy. If you know that one of their children is a girl, what is the probability that the other child is also a girl?

3. Given a group of 25 people, what is the probability that at least two of them share the same birthday (day and month, not necessarily year)?