1. Suppose you have 100 bottles of soda, and you know that one of them has been poisoned, but not which one. The poison is so strong that drinking any amount of the poisoned soda is deadly, but it takes 24 hours to take effect. You also have 100 rats, and your only way of testing to see if the soda is poisoned is to give it to the rats to drink. You’re allowed to give each rat a mixture consisting of soda from any number of the bottles. What is the minimum number of rats you can use to be sure that you find the poisoned bottle?

2. Suppose you have 100 coins and a simple balance scale. You know that one of the coins is counterfeit and that counterfeit coins weigh less than genuine coins. Using only the scale, what is the minimum number of weighings you need to do in order to find the counterfeit coin? For an extra challenge, suppose you don’t know whether counterfeit coins are lighter or heavier than genuine coins. How does this change your answer?
3. A certain pirate ship has a crew of 101 pirates. They are ranked, in order, from the captain, to the first mate, to the second mate, all the way down to the hundredth mate. The pirates have a particular system for dividing up treasure. When they find some treasure, the captain proposes an amount to be given to each pirate (including himself), and the then the pirates vote. If half or more of the crew (again, including the captain) agree, then the treasure is divided up as the captain proposed. Otherwise, the captain has to walk the plank, and the first mate becomes the new captain. He must now make a proposal, and another vote will take place. Suppose the pirates find a chest containing a thousand gold coins. What is the maximum number of gold coins the captain can keep for himself and still ensure that his proposal will pass?

4. Suppose that you have a checkerboard that is only 2 squares tall, but is a hundred squares long. You also have a collection of dominoes, each of which is exactly the size of two squares of the checkerboard. How many different ways can you tile the checkerboard with dominoes so that no squares are showing? For an extra challenge, answer the same question for a 3 x 100 checkerboard. What about a 4 x 100? What about a general m x n checkerboard?