Formal Logic

1. What is a statement (or proposition)? Give an example of a sentence that is a statement and an example of a sentence that is not a statement.

2. Create a truth table for the negation, conjunction, and disjunction of statements $p$ and $q$.

3. Create a truth table for the conditional $p \rightarrow q$, and prove using a truth table that $p \rightarrow q$ is logically equivalent to $(\neg p) \lor q$.

4. Prove De Morgan’s law: if $p$ and $q$ are statements, then $\neg(p \lor q)$ is logically equivalent to $\neg p \land \neg q$.

5. What is a proof by contradiction? Mini-challenge: use proof by contradiction to show that there is no biggest integer (That is, show that there is no integer $N$ such that $N \geq n$ for every integer $n$).

The Island of Knights and Knaves

Solve each puzzle and justify your answers. Assume each of A and B (and C, if present) is either a knight or a knave. Remember that knights always tell the truth and knaves always lie.

1. Suppose A says:

   A: Either I am a knave or B is a knight.

   What are A and B?
2. Suppose A says:

   A: I am a knave and B is a knight.

   What are A and B?

3. Suppose A and B say:

   A: Both of us are knights.
   B: A is lying and $2 + 2 = 4$.

   What are A and B?

4. Suppose A says:

   A: If B is a knight, then I am a knave.

   What are A and B?

5. Suppose A and B say:

   A: B is a knight.
   B: If A is a knight, then C is a knight.

   What are A, B, and C?